2023 泛函分析及空间理论天元暑期研讨班

Concentration day: free probability

Friday, July 28, 2023   Zheng Xin Building, Room 21

Spectrum of Laplacian matrix associated with large random elliptic matrices

Zhi YIN（尹智），*Central South University*   9:30

We study the limit eigenvalue distribution of the Laplacian matrix of a random elliptic matrix. Under some mild assumptions, we show that the empirical spectral distribution of the Laplacian matrix converges to a deterministic probability distribution. The limit measure can be interpreted as the Brown measure of the sum of an elliptic operator and a normal operator with a Gaussian distribution freely independent from the elliptic operator. This is a joint work with Sean O'Rourke and Ping Zhong.

The spectrum of local random Hamiltonians

Liang ZHAO（赵亮），*Harbin Institute of Technology*   11:00

The spectrum of a local random Hamiltonian can be represented generically by the so-called ε-free convolution of the probability distributions of its local terms. In this talk, I will introduce the basic definitions and properties of ε-freeness. Further, I will show an isomorphism between the set of ε-noncrossing partitions and permutations to study its spectrum. Moreover, we derive some lower and upper bounds for the maximal eigenvalue of the Hamiltonian.

Free probability and heat flow on GAF

Ching Wei HO（何政卫），*Academia Sinica*   14:30

Two fundamental distributions in free probability are the semicircular law and the circular law. An elementary computation shows that the push forward of the circular law by the function $z \mapsto 2\mathrm{Re}(z)$ is the semicircular law. We then ask: do we have a random matrix or finite-N counterpart of this simple push-forward property? In this talk, I will introduce a conjecture of Hall and I that this property can be resembled by the "heat flow (differential) operator" applied to the characteristic polynomial of the Ginibre ensemble. While the conjecture has not been solved, I will talk about a closely related rigorous result of mine with Hall, Jalowy, Kabluchko about applying the heat flow operator on GAF, whose zero distribution is "uniform" on the complex plane with spacing of order 1. If time permits, I will say a few words about the relation of heat flow on GAF and the metaplectic representation of $SU(1,1)$.

Rank inequality done by free probability

Sheng YIN（尹晟），Baylor University　　　　　　　　　　　　　　　　16:00

In linear algebra it is well-known that the matrix rank is subadditive, i.e.,

$$\mathrm{rank}(A + B) \leq \mathrm{rank}(A) + \mathrm{rank}(B)$$

for matrices A, B. One can further ask for any polynomial p other than $x + y$, what upper bound for $\mathrm{rank}(p(A, B))$ one can hope for matrices A, B? Moreover, can it be dimensionless optimal in some sense?

In this talk, we will provide some answers to the above question based on a recent joint-work with Octavio Arizmendi, Guillaume Cébron and Roland Speicher. More precisely, our answers are based on a universality property of freely independent random variables with respect to the von Neumann rank. This universality property was revealed along our effort to built a machinery to calculate the atoms/ranks of polynomials in freely independent random variables. In particular, we provided explicit formulae for the commutator $p = xy - xy$ and the anticommutator $p = xy + yx$, which can yield an dimensionless optimal upper bound for their matrix rank $\mathrm{rank}(p(A, B))$.