QUANTITATIVE ESTIMATES FOR THE FUNCTIONAL CALCULUS ON THE SCHATTEN p-CLASSES, $0 < p < \infty$.

FEDOR SUKOCHEV (UNIVERSITY OF NEW SOUTH WALES, AUSTRALIA)

Abstract. We focus on the Lipschitz estimates for functional calculus in Schatten S_p-classes.

We briefly recall Lipschitz estimates for Schatten classes, S_p, $1 < p < \infty$ and $p = 1$. Here, we mainly focus on the result by Potapov and Sukochev (Acta Mathematica, 2011) which, in this setting, resolved a famous problem set by Mark Krein in 1953. We also recall a familiar result by Peller concerning operator Lipschitz functions in the setting $p = 1$ and $p = \infty$ that $f \in \dot{B}^1_{\infty,1}$ is sufficient for a function to be Lipschitz in S_1 and S_∞.

The main object of this lecture is devoted to the recent joint result of McDonald and Sukochev (*Lipschitz estimates in quasi-Banach Schatten ideals.* Math. Ann. **383** (2022), no. 1–2, 571–619.) which is devoted to the the similar problem, but in the setting of quasi-normed Schatten classes, S_p, $0 < p < 1$.

The quasi- Banach range $0 < p < 1$ is by comparison poorly understood. Using (somewhat unexpected) techniques from wavelet analysis, we prove that Lipschitz functions belonging to the homogeneous Besov class $\dot{B}^{\frac{1}{p}}_{\frac{p}{1-p},p}(\mathbb{R})$ obey the estimate

$$\|f(A) - f(B)\|_p \leq C_p \left(\|f'\|_{L_\infty(\mathbb{R})} + \|f\|_{\dot{B}^{\frac{1}{p}}_{\frac{p}{1-p},p}(\mathbb{R})} \right) \|A - B\|_p$$

for all bounded self-adjoint operators A and B with $A - B \in \mathcal{L}_p$. In the case $p = 1$, our methods actually recover and provide a new perspective on the Peller's result. In addition, we prove the surprising fact that non-constant periodic functions on $\mathbb{R}$ are not Lipschitz in $\mathcal{L}_p$ for any $0 < p < 1$. This implies the existence of counterexamples to a familiar 1991 conjecture of Peller that $f \in \dot{B}^{1/p}_{\infty,p}(\mathbb{R})$ is sufficient for f to be Lipschitz in S_p.