The Role of the Transpose in Free Probability: the partial transpose of R-cyclic operators.

James A. Mingo

Like tensor independence, free independence gives us rules for doing calculations. With random matrix models, we usually need tensor independence of the entries and some kind of group invariance of the joint distribution of the entries to get the asymptotic) freeness necessary to apply the tools of free probability.

A few years ago, Mihai Popa and I found that the transpose also produces asymptotic freeness, i.e. a matrix could be asymptotically free from its own transpose. Since that we have expanded this work to the case of the partial transposes that arise in quantum information theory.

In this talk I will explain what happens when one transposes certain R-cyclic operators.

Regularity properties of non-commutative matrices and rational functions: the absence of zero divisors

Roland Speicher

We consider non-commutative rational functions as well as matrices in polynomials in non-commuting variables in two settings: in an algebraic context, the variables are formal variables, and their rational functions generate the "free field"; in an analytic context, the variables are given by operators from a finite von Neumann algebra and the question of rational functions is treated within the affiliated unbounded operators. In recent joint work with Tobias Mai and Sheng Yin we have shown that for a "good" class of operators -- namely those for which the free entropy dimension is maximal -- the analytic and the algebraic theory are isomorphic. I will explain the context and ideas of these results as well as the connection with questions on zero divisors.

In the follow-up talk by Sheng Yin the relation of all this with the Atiyah property will be addressed.

Regularity properties of non-commutative matrices and rational functions: Atiyah properties

Sheng Yin

In this talk, we continue the story concerning the zero divisor problem for matrices in polynomials evaluated at some good operators. Actually, we want to show that if we translate the zero divisor problem as the invertibility over the affiliated unbounded operators, then this question is equivalent to some Atiyah property. Moreover, with the help of non-commutative rational functions, we find a new fact about these ranks in Atiyah properties, that is, these ranks are actually equal to some algebraic ranks in our setting. Then these properties can answer questions on some regularity properties of the distribution of matrices and rational functions in certain non-commuting variables, and thus also in corresponding random matrices.

Interpolation of Haagerup noncommutative Hardy spaces

Turdebek N. Bekjan

Let M be a \sigma-finite von Neumann algebra, equipped with a normal faithful state \phi, and let A be maximal subdiagonal algebra of M. We prove Stein-Weiss type interpolation theorem of Haagerup noncommutative H_p-spaces associated with A.

Noncommutative good-λ inequalities

Yong Jiao

We propose a novel approach in noncommutative probability, which can be regarded as an analogue of good-λ inequalities from the classical case due to Burkholder and Gundy (Acta Math 124: 249-304, 1970). This resolves a longstanding open problem in noncommutative realm. Using this technique, we present new proofs of noncommutative Burkholder-Gundy inequalities, Stein's inequality, Doob's inequality and L^p-bounds for martingale transforms; all the constants obtained are of optimal orders. The approach also allows us to investigate the noncommutative analogues of decoupling techniques and, in particular, to obtain new estimates for noncommutative martingales with tangent difference sequences and sums of tangent positive operators. These in turn yield an enhanced version of Doob's maximal inequality for adapted sequences and a sharp estimate for a certain class of Schur multipliers. We expect the method to be useful in other settings as well. This is a joint work with A. Oskekowski and L. Wu.

Noncommutative Poisson process with applications to Banach space geometry

Dimitry Zanin

In the classical probability theory, Kruglov operator (an integration with respect to the Poisson process) possesses a remarkable property: it maps disjointly supported functions to independent random variables.

We extend the notion of Kruglov operator to noncommutative von Neumann algebras. For finite von Neumann algebra, the construction is elementary. Surprisingly, there is a construction of Kruglov operator on semifinite von Neumann algebras.

Having this operator at hands, we provide embedding theory of the Banach ideals (in $B(H)$) to L_p-spaces on a hyperfinite factor.

The PPT square conjecture holds generically for some classes of independent states

Zhi Yin

Let $|\psi\rangle\langle\psi|$ be a random pure state on $\mathbb{C}^{d^2} \otimes \mathbb{C}^{s}$, where ψ is a random unit vector uniformly distributed on the sphere in $\mathbb{C}^{d^2} \otimes \mathbb{C}^{s}$. Let ρ_1 be random induced states $\rho_1 = \mathrm{Tr}_s (|\psi\rangle\langle\psi|)$ whose distribution is $\mu_{d^2,s}$; and let ρ_2 be random induced states following the same distribution $\mu_{d^2,s}$ independent from ρ_1. Let ρ be a random state induced by the entanglement swapping of ρ_1 and ρ_2. We show that the empirical spectrum of $\rho - \mathrm{id}/d^2$ converges almost surely to the Marcenko-Pastur law with parameter c^2 as $d \to \infty$ and $s/d \to c$. As an application, we prove that the state ρ is separable generically if ρ_1, ρ_2 are PPT entangled. This is a joint work with Benoit Collins (Kyoto) and Ping Zhong (Waterloo).