Integrability in Random Matrix Theory.

Applications to quantum transport and quantum chaos

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Education

- Department of Theoretical Nuclear Physics
Moscow State Engineering-Physics Institute, Russia

(1998) Diploma work: "*Emission of a charge moving uniformly parallel to the surface of a non-stationary medium.*"
V.Al.Osipov, M.I.Ryazanov, *Laser Physics* **8** (1998) 1007

- PhD in Physical and Mathematical Sciences
N.N.Semenov Institute of Chemical Physics of the Russian Academy of Sciences, Russia

(2003) Thesis: "p-Adic models of ultrametric diusion and their application to the description of protein conformational dynamics"
V.A.Avetisov V.A., A.Kh.Bikulov, S.V.Kozyrev, **V.Al.Osipov**, *J.Phys.A* **35** (2002) 177

Working Experience, research

Languages: Russian, English, German, Hebrew

- *1998-2005 Engineer, Researcher at*
  Condensed Matter Physics, Institute of Chemical Physics of the Russian Academy of Sciences (Russia)
- *2005-2008 Postdoctoral Associate at*
  Applied Math. H.I.T. – Holon Institute of Technology (Israel)
- *2008-2011 Research Associate at*
  Physics, Duisburg – Essen University (Germany)
- *2011-2014 Research Associate at*
  Institute of Theoretical Physics, Cologne University (Germany),
  Excellence university since 2012
- *2014-2015 Distinguished position at*
  Physics, Duisburg – Essen University (Germany)
- *2015-2017 Postdoctoral Associate at*
  Chemical Physics, Lund University (Sweden)
- *2018-2019 Associate Specialist at*
  Chemistry, University of California, Irvine (USA)
- *2019- Research Associate at*

  Physics, H.I.T. – Holon Institute of Technology (Israel)

Research Interests: Mathematical and Theoretical Physics

- **Mathematical Physics and applications**
  - **Ultrametrisity and models of ultrametric diffusion**
  - **Theory of Random Matrices**
  - **Quantum Chaos**
  - **Quantum Systems with Disorder**
- **Theory of nonlinear optical systems and Chemical Physics**
  - **Nonlinear spectroscopy**
  - **Nanodevices**

Integrability in Random Matrix Theory.

Applications to quantum transport and quantum chaos

V.Al.Osipov, E.Kanzieper, “Are bosonic replicas faulty?” *Phys.Rev.Let.* **99** (2007) 050602

V.Al.Osipov, E.Kanzieper, “Integrable theory of quantum transport in chaotic cavities”, *Phys.Rev.Let.* **101** (2008) 176804

V.Al.Osipov, E.Kanzieper, “Statistics of thermal to shot noise crossover in chaotic cavities”, *J.Phys.A:Math.Theor.* **42** (2009) 475101

V.Al.Osipov, E.Kanzieper, “Correlations of RMT characteristic polynomials and integrability: Random Hermitian matrices”, *Annals of Physics* **325** (2010) 2251

V.Al.Osipov, H.-J.Sommers, K.Zyczkowski, “Random Bures matrices and the distribution of their purity”, *J.Phys.A:Math.Theor.* **43** (2010) 055302

R.Riser, **V.Al.Osipov**, E.Kanzieper, “Power-spectrum of long eigenlevel sequences in quantum chaology”, *Phys.Rev.Let.* **118** (2017) 204101

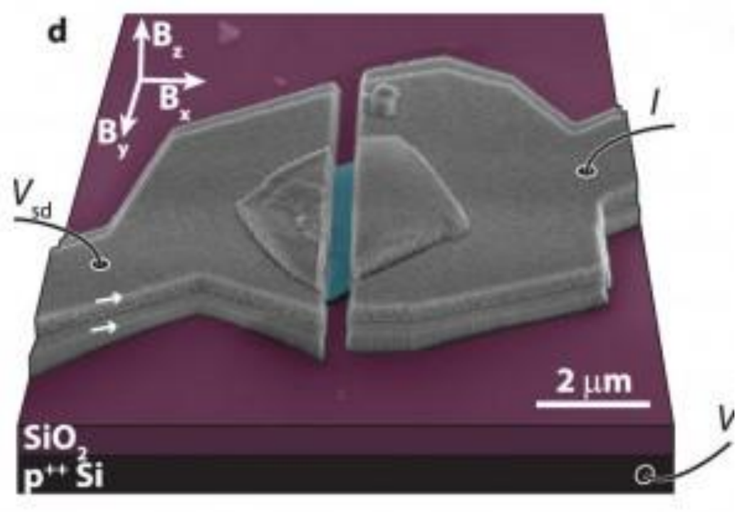
R.Riser, **V.Al.Osipov**, E.Kanzieper, “Nonperturbative theory of power spectrum in complex systems”, *Annals of Physics* **413** (2020) 168065

Integrability in Random Matrix Theory.
Applications to quantum transport and quantum chaos

- Random Matrices for description of quantum transport in chaotic cavities.
- The Integrable theory of Random Matrices
- Other applications

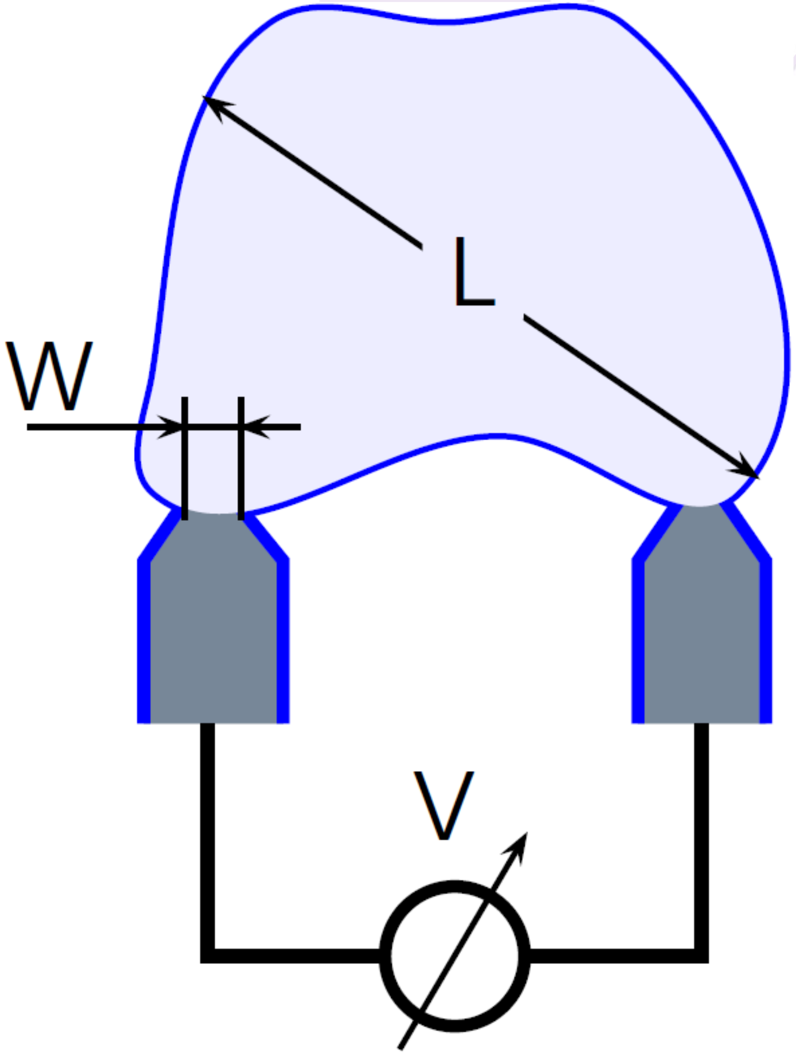
Mesoscopic physics

- The system geometric scales is between macroscopic and microscopic (10^{-6} m);
- Both the quantum and statistical descriptions are required;
- Significant fluctuations of the observables requires mean value, moments and distributions;
- Conductance **G** is the easiest to measure;
- A universal method of description is of interest.



$$\Delta I = G\,\Delta V$$

Quantum transport in chaotic cavities

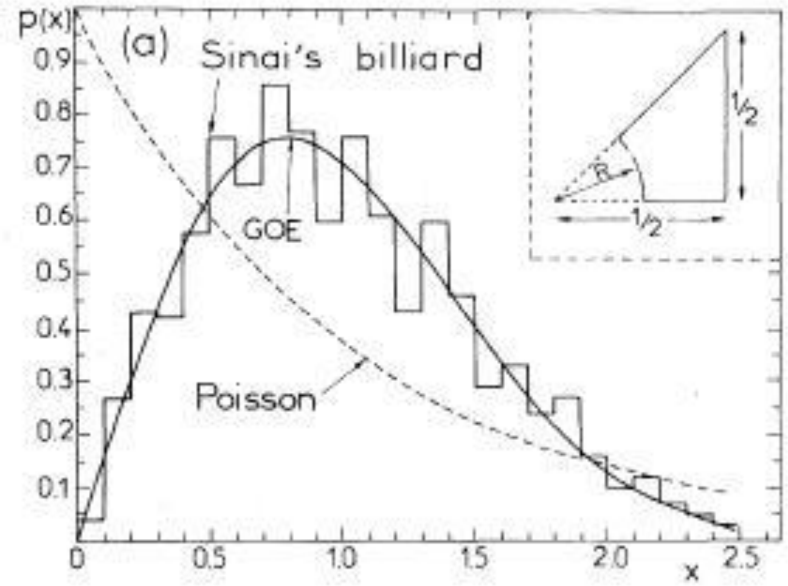


$$H_{\mathrm{tot}} = \underbrace{\sum_{\alpha=1}^{N_L+N_R} \chi_\alpha^\dagger \varepsilon_F \chi_\alpha}_{\text{leads}} + \underbrace{\sum_{k,l=1}^{M} \psi_k^\dagger \mathcal{H}_{kl} \psi_l}_{\text{cavity}}$$

$$+ \underbrace{\sum_{k=1}^{M} \sum_{\alpha=1}^{N_L+N_R} \left(\psi_k^\dagger \mathcal{W}_{k\alpha} \chi_\alpha + \chi_\alpha^\dagger \mathcal{W}_{k\alpha}^* \psi_k \right)}_{\text{coupling}}$$

- Single electron on the Fermi surface without losses (infinite potential walls)
- Broken time-reversal symmetry: the Hamiltonian is Hermitian
- The cavity is chaotic (the classical billiard of the same shape is chaotic)
- Universal regime: Electron dwell time >> Ehrenfest time (W<<L)

Quantum transport in chaotic cavities
Random matrix



Nearest neighbor density for a (desymmetrized) Sinai billiard, showing that this **distribution agrees perfectly well with the Wigner surmise** (1956)

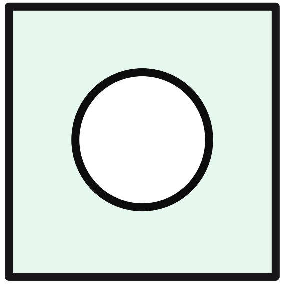
$$p(x) = \frac{\pi}{2} x e^{-\frac{\pi}{4}x^2}$$

valid for the Gaussian Orthogonal Ensemble (solid line).

- **Bohigas-Giannoni-Schmit** conjecture (1984):

Hamiltonian of a chaotic system can be replaced by a random matrix of the proper symmetry

Sinai billiard



N x N random Gaussian Hermitian matrix (GOE) $\mathcal{H}^T = \mathcal{H}$

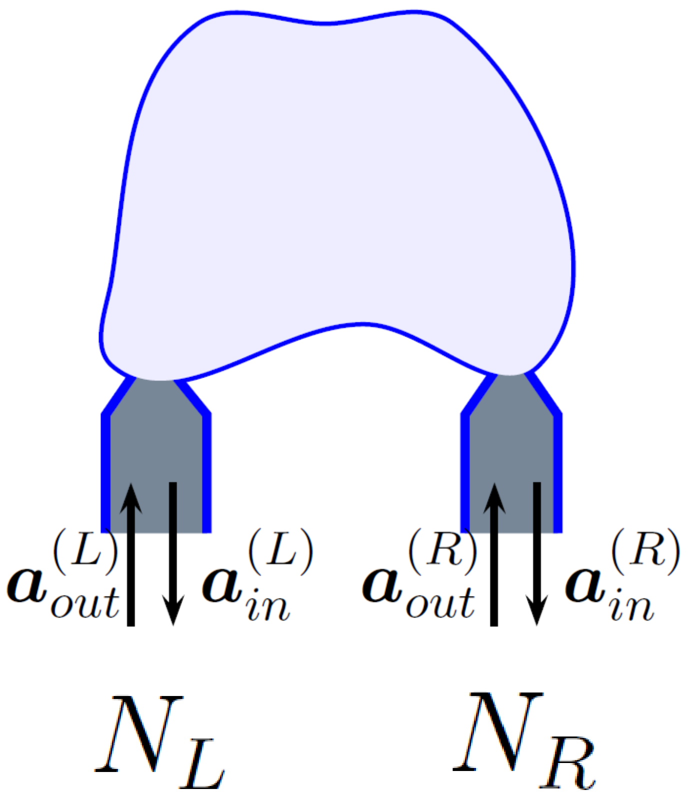
$$dP(\mathcal{H}) \propto e^{-\frac{N}{4}\mathrm{Tr}\,\mathcal{H}^2} d\mathcal{H} = \prod_{i,j}^{N} e^{-\frac{N}{4}\mathcal{H}_{i,j}^2} d\mathcal{H}_{i,j}$$

The distribution of the nearest eigenlevel spacings:

$$p(x) = \left\langle \frac{1}{N} \sum_{j=1}^{N-1} \delta\big(x - (E_{j+1} - E_j)\big) \right\rangle$$

There are many other works were done to support the conjecture.

Quantum transport in chaotic cavities
Random scattering matrix



- Scattering matrix connects the amplitudes of incoming quantum waves with the amplitudes of outcoming waves; r – reflection part, t – transmission part

$$\begin{pmatrix} \boldsymbol{a}_{out}^{(L)} \\ \boldsymbol{a}_{out}^{(R)} \end{pmatrix} = \mathcal{S} \begin{pmatrix} \boldsymbol{a}_{in}^{(L)} \\ \boldsymbol{a}_{in}^{(R)} \end{pmatrix} \qquad \mathcal{S} = \begin{pmatrix} r & t \\ t' & r' \end{pmatrix} \qquad \mathcal{S}\mathcal{S}^\dagger = \mathbb{1}$$

$$\mathcal{S} = \mathbb{1} - 2i\pi \mathcal{W}^\dagger (\varepsilon_F - \mathcal{H} + i\pi \mathcal{W}\mathcal{W}^\dagger)^{-1} \mathcal{W}$$

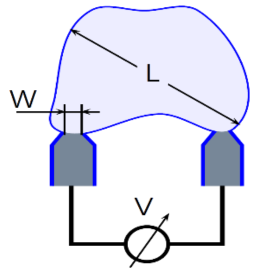
- Conductance is transmission (Landauer 1958)

$$g = \mathrm{Tr}\, tt^\dagger = \sum_{j=1}^{n} T_j \qquad n = \min\{ N_L, N_R \}$$

- Bohigas-Giannoni-Schmit conjecture (1984):
  The Hamiltonian of a chaotic cavity is replaced by
  a random matrix drawn from GUE: $\mathcal{H}^\dagger = \mathcal{H}$

- Ballistic point contacts: scattering matrix is a random unitary matrix
  (Blumel, Smilansky 1990), circular unitary ensemble CUE

Quantum transport in chaotic cavities
Random matrix integrals



The observable: conductance $g = \mathrm{Tr}\ tt^\dagger = \sum_{j=1}^{n} T_j$

The sought quantity is the moment generation function (Laplace transform of the conductance density)

$$\rho(g) \equiv \left\langle \delta\left(g - \sum_{j=1}^{n} T_j\right)\right\rangle = \mathcal{L}^{-1}[\mathcal{F}_n(z)](g)$$

Integrating out the angular degrees of freedom transforms $F(z)$ to the n-fold integral over the transmission coefficients

$$\mathcal{F}_n(z) = \langle \exp(-zg) \rangle_{\mathcal{S}\in \mathrm{CUE}(N_L+N_R)}$$

$$n = \min\{N_L, N_R\} \qquad \nu = |N_L - N_R|$$

$$\mathcal{F}_n(z) = c_n^{-1} \prod_{j=1}^{n} \left[\int_0^1 e^{-zT_j} T_j^{\nu} dT_j\right] \cdot \prod_{i<j} (T_i - T_j)^2$$

Squared Vandermonde determinant

Moment generation function:

$$\mathcal{F}_n(z) = \sum_{\ell=1}^{\infty} (-1)^\ell \mathfrak{m}_\ell \frac{z^\ell}{\ell!}$$

Cumulant generation function:

$$\log \mathcal{F}_n(z) = \sum_{\ell=1}^{\infty} (-1)^\ell \kappa_\ell \frac{z^\ell}{\ell!}$$

Integrable theory of random matrices

Theory of τ-function

$$\mathcal{F}_n(z) = c_n^{-1} \prod_{j=1}^{n} \left[\int_0^1 e^{-zT_j} T_j^{\nu} dT_j \right] \cdot \Delta_n^2 \{ \boldsymbol{T} \}$$

Vandermonde determinant

$$\Delta_n \{ \boldsymbol{T} \} \equiv \prod_{i<j}^{n} |T_i - T_j| = \det \begin{bmatrix} 1 & T_1 & T_1^2 & \dots & T_1^{n-1} \\ 1 & T_2 & T_2^2 & \dots & T_2^{n-1} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & T_n & T_n^2 & \dots & T_n^{n-1} \end{bmatrix}$$

The matrix integral $\frac{1}{n!} \int_{\mathcal{D}^n} \Delta_n^2 \{ \boldsymbol{T} \} \cdot \prod_{j=1}^{n} \exp \left[-V[T_j] + \sum_{m=1}^{\infty} u_m T_j^m \right] dT_j$

is a τ –function that satisfies the Toda-lattice hierarchy and the Kadomtsev-Petviasvili hierarchies.

Adler, van Moerbeke (1995)

Integrable theory of random matrices

Theory of τ-function

The matrix integral

$$\frac{1}{n!}\int_{\mathcal{D}^n} \Delta_n^2\{\boldsymbol{T}\}\cdot\prod_{j=1}^{n}\exp\left[-V[T_j]+\sum_{m=1}^{\infty}u_m T_j^m\right]dT_j$$

is a τ –function that satisfies the Toda-lattice hierarchy and the Kadomtsev-Petviasvili hierarchies.

Adler, van Moerbeke (1995)

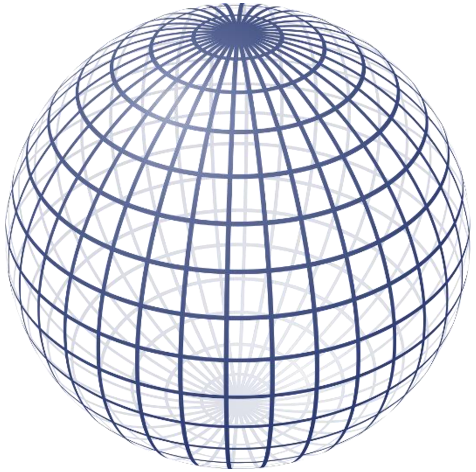
$$\tau\{u_m\}_{m=1}^{\infty}$$

τ - function is a function, which depends on an infinite number of parameters and satisfies an infinite number of relations.

R.Hiroto, M.Sato (1981)

Complete chaos

Infinite number of integrals of motion and infinite number of degrees of freedom.



Complete integrability

The number of degrees of freedom coincide with the number of integrals of motion.

Integrable theory of random matrices

Theory of τ-function, Toda lattice

$$\mathcal{F}_n(z) = c_n^{-1} \prod_{j=1}^{n} \left[\int_0^1 e^{-zT_j} T_j^{\nu} dT_j \right] \cdot \Delta_n^2 \{ \boldsymbol{T} \}$$

Toda lattice hierarchy, the first equation:

$$\mathcal{F}_n''(z) = \mathcal{F}_n(z) + \mathrm{var}(g) \frac{\mathcal{F}_{n-1}(z)\mathcal{F}_{n+1}(z)}{\mathcal{F}_n(z)}$$

Initial conditions:

$$\mathcal{F}_0(z) \equiv 1;$$

$$\mathcal{F}_1(z) = \frac{(\nu+1)!}{z^{\nu+1}} \left(1 - e^{-z} \sum_{\ell=0}^{\nu} \frac{z^{\ell}}{\ell!} \right)$$

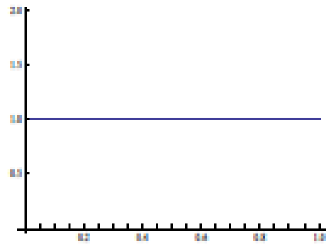
One can calculate the conductance density for small n (number of open channels)

$$\rho(g) \equiv \left\langle \delta \left(g - \sum_{j=1}^{n} T_j \right) \right\rangle_{\boldsymbol{T}} = \mathcal{L}^{-1} \left[\mathcal{F}_n(z) \right](g)$$

Quantum transport in chaotic cavities

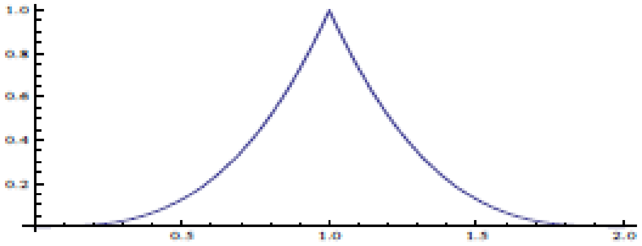
Theory of τ-function, Toda lattice

$n=1$



$$\rho_1(g) = 1$$

$n=2$



$$\rho_2(g) = g^3 + 2(4-6g+3g^2-g^3)\Theta(g-1)$$

$n=3$

$$\rho_3(g) = \frac{g^8}{28} + \frac{(g-2)^4}{7}(156-144g+80g^2-8g^3+g^4)\Theta(g-2) - \frac{(g-2)^4}{7}(309-228g+62g^2-4g^3+g^4)\Theta(g-1)$$

Geometry: two leads with N_L and N_R numbers of propagating modes;
Number of open channels: $n = \min\{N_L, N_R\}$;

Asymmetry parameter: $\nu = |N_L - N_R| = 0$.

n/2 –is the mean value of g.
The Gaussian approximation
is only valid for |g-n/2|<n/4

Quantum transport in chaotic cavities

Theory of τ-function, Kadomtsev-Petviashvili

$$\mathcal{F}_n(z,\{\boldsymbol{u}\}) = c_n^{-1} \prod_{j=1}^{n} \left[\int_0^1 e^{-zT_j + \sum_{m=2}^{\infty} u_m T_j^m} T_j^{\nu} dT_j \right] \cdot \Delta_n^2\{\boldsymbol{T}\}$$

The first equation of Kadomtsev-Petviashvili hierarchy (KP-equation):

$$\left(\frac{\partial^4}{\partial z^4} + 3\frac{\partial^2}{\partial u_2^2} + 4\frac{\partial^2}{\partial z \partial u_3} \right) \log \mathcal{F}_n + 6 \left(\frac{\partial^2}{\partial z^2} \log \mathcal{F}_n \right)^2 = 0$$

Projection onto the hyperplane $\boldsymbol{u}=0$

The projections of the derivatives $\left.\frac{\partial}{\partial u_2} \log \mathcal{F}_n \right|_{\boldsymbol{u}=0}$ $\left.\frac{\partial}{\partial u_3} \log \mathcal{F}_n \right|_{\boldsymbol{u}=0}$ are unknown.

The missing block is the Virasoro constraints

Quantum transport in chaotic cavities
Virasoro constraints

$$\mathcal{F}_n(z,\{\boldsymbol{u}\}) = c_n^{-1} \prod_{j=1}^{n} \left[\int_0^1 e^{-zT_j + \sum_{m=2}^{\infty} u_m T_j^m} T_j^{\nu} dT_j \right] \cdot \Delta_n^2\{\boldsymbol{T}\}$$

The Virasoro constraints

$$\frac{\delta}{\delta\epsilon} \left(\mathcal{F}_n(z,\{\boldsymbol{u}\}) \Big|_{T_j \to T_j + \delta\epsilon\, T_j^{q+1}(1-T_j)} \right) = 0$$

$$\hat{\mathcal{L}}_q \mathcal{F}_n(z,\{\boldsymbol{u}\}) = 0$$

$$q = 0, 1, \ldots$$

$$[\mathcal{L}_p, \mathcal{L}_q] = (p-q)\mathcal{L}_{p+q}$$

The integral is invariant with respect to transformation of the integration measure (the transformation is zero at the boundaries of the integration domain).

$$\hat{\mathcal{L}}_0 = \sum_{m=2}^{\infty} m u_m \left(\frac{\partial}{\partial u_{m+1}} - \frac{\partial}{\partial u_m} \right) - 2z \left(\frac{\partial}{\partial u_2} + \frac{\partial}{\partial z} \right) - (2n+\nu)\frac{\partial}{\partial z} - n(n+\nu)$$

Quantum transport in chaotic cavities
Conductance cumulants

$$\mathcal{F}_n(z, \{\boldsymbol{u}\}) = c_n^{-1} \prod_{j=1}^{n} \left[\int_0^1 e^{-zT_j + \sum_{m=2}^{\infty} u_m T_j^m} T_j^{\nu} dT_j \right] \cdot \Delta_n^2 \{\boldsymbol{T}\}$$

The Virasoro constraints + Kadomtsev-Petviashvili projection onto u=0 give rise to the Painleve V

$$\mathcal{F}_n(z, \{\boldsymbol{u}\})\Big|_{\boldsymbol{u}=0} = \exp\left(\int_0^z \frac{\sigma_V(x) - n(n+\nu)}{x} dx \right)$$

Requirement of convergence: $\sigma_V(0) = n(n+\nu)$

Painleve V: $(x\sigma_V'')^2 + \left[\sigma_V - x\sigma_V' + 2(\sigma_V')^2 + (2n+\nu)\sigma_V'\right]^2 + 4(\sigma_V')^2(\sigma_V' + n)(\sigma_V' + n + \nu)^2 = 0$

Moment generation function:

$$\mathcal{F}_n(z) = \sum_{\ell=1}^{\infty} (-1)^{\ell} \mathfrak{m}_{\ell} \frac{z^{\ell}}{\ell!}$$

Cumulant generation function:

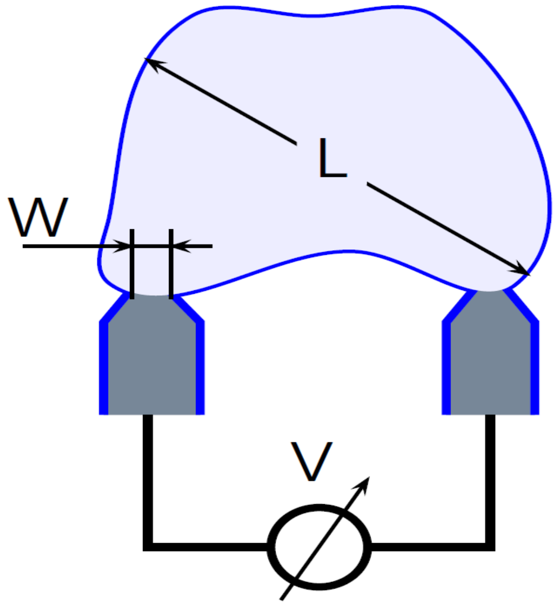
$$\log \mathcal{F}_n(z) = \sum_{\ell=1}^{\infty} (-1)^{\ell} \kappa_{\ell} \frac{z^{\ell}}{\ell!}$$

Quantum transport in chaotic cavities

Conductance cumulants

Geometry of two leads with N_L and N_R number of open modes
Number of open channels: $n = \min\{N_L, N_R\}$
Asymmetry parameter: $\nu = |N_L - N_R|$

$$j \geq 2: \qquad [(2n+\nu)^2 - j^2](j+1)\kappa_{j+1} = 2\sum_{\ell=0}^{j-1} \frac{j!(3\ell+1)(j-\ell)^2}{\ell!(j-\ell)!}\kappa_{\ell+1}\kappa_{j-\ell} - (2n+\nu)(2j-1)\,j\kappa_j - j(j-1)(j-2)\,\kappa_{j-1}$$



Mean value $$\kappa_1 = \frac{n(n+\nu)}{2n+\nu}$$

Variance $$\kappa_2 = \frac{1}{(2n+\nu)^2 - 1}\kappa_1^2$$

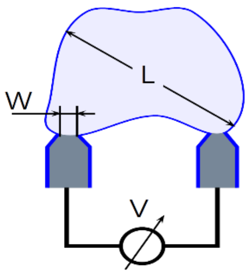
Skewness $$\kappa_3 = -\frac{2\nu^2}{((2n+\nu)^2-1)((2n+\nu)^2-4)}\kappa_1^2 < 0$$

$n \gg 1 \quad \nu = 0$

$$\kappa_j = \underbrace{\frac{n}{2}\delta_{j,1} + \frac{1}{16}\delta_{j,2}}_{\text{Gaussian part}} + \frac{1+(-1)^j}{8}\frac{(2j-1)!}{(4n)^{2j}}\left[1 + \frac{j(3j^2-1)}{8n^2} + \mathcal{O}\left(\frac{1}{n^4}\right)\right]$$

Integrability in Random Matrix Theory.

Applic … uantum chaos



$$\mathcal{F}_n(z) = \langle \exp(-zg) \rangle_{\mathcal{S}\in \mathrm{CUE}(2n+\nu)}$$

$$n = \min\{N_L, N_R\} \qquad \nu = |N_L - N_R|$$

$$\mathcal{F}(z) = c_n^{-1} \prod_{j=1}^{n} \left[\int_0^1 e^{-zT_j} T_j^{\nu} dT_j \right] \cdot \prod_{i<j} (T_i - T_j)^2$$

Integrable theory

Toda Lattice hierarchy:

$$\mathcal{F}_n(z)\mathcal{F}_n''(z) - (\mathcal{F}_n(z))^2 = \mathrm{var}(g)\mathcal{F}_{n-1}(z)\mathcal{F}_{n+1}(z)$$

Representation in terms of Painleve V transcendent:

$$\mathcal{F}_n(z) = \exp\left(\int_0^z \frac{\sigma_V(x) - n(n+\nu)}{x} dx \right) \qquad \sigma_V(0) = n(n+\nu)$$

$$(x\sigma_V'')^2 + \left[\sigma_V - x\sigma_V' + 2(\sigma_V')^2 + (2n+\nu)\sigma_V'\right]^2 + 4(\sigma_V')^2(\sigma_V' + n)(\sigma_V' + n + \nu)^2 = 0$$

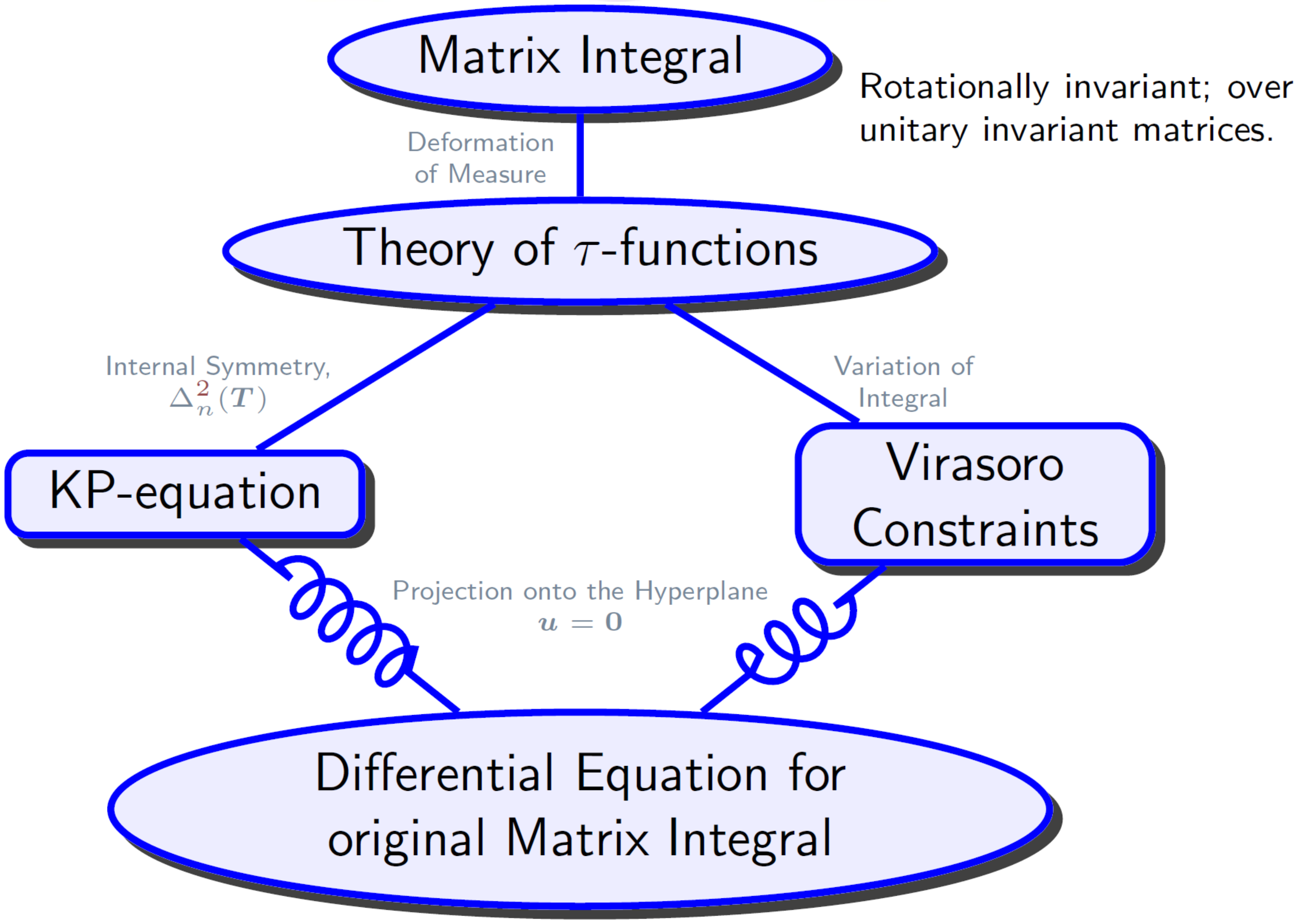
V.Al.Osipov, E.Kanzieper, "Integrable theory of quantum transport in chaotic cavities", *Phys.Rev.Let.* **101** (2008) 176804

Integrability theory for Random Matrices

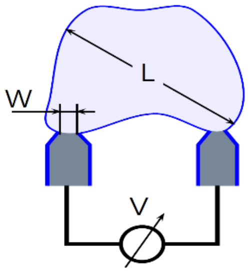
Often the matrix integral is represented in terms of (one of six) Painleve equation due to KP-hierarchy.

The other hierarches (TL) give rise to additional relations

Sometimes the Virasoro constraints are not resolvable.



Quantum transport in chaotic cavities
Conductance-shot-noise power joint cumulants



-- The realistic Hamiltonian of the cavity is replaced by a random Hermitian matrix (Bohigas-Giannoni-Schmit conjecture). The scattering matrix is drawn from CUE.

-- The transport characteristics are calculated as an average of the observable over ensemble of random matrices.

The observable: conductance $g = \sum_{j=1}^{n} T_j$ and the shot-noise power $p = \eta \sum_{j=1}^{n} T_j(1 - T_j)$

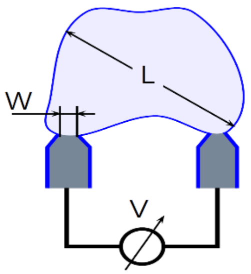
The sought quantity is the joint moment generation function

$$\mathcal{F}_n(z, w) = \langle \exp\left(-zg\right) \exp\left(-wp\right) \rangle_{\mathcal{S} \in \mathrm{CUE}(2n+\nu)}$$

$$\log \mathcal{F}_n(z, w) = \sum_{\ell=1}^{\infty} (-1)^{\ell+m} \kappa_{\ell,m} \frac{z^\ell w^m}{\ell! m!}$$

Quantum transport in chaotic cavities

Conductance-shot-noise power joint cumulants



conductance $g = \sum_{j=1}^{n} T_j$ and the shot-noise power $p = \eta \sum_{j=1}^{n} T_j(1 - T_j)$

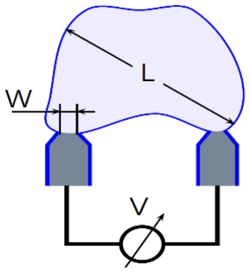
$$\mathcal{F}_n(z,w) \propto \prod_{j=1}^{n} \left[\int_0^1 e^{-zT_j - w\eta T_j(1-T_j)} T_j^{\nu} dT_j \right] \cdot \Delta_n^2 \{ \boldsymbol{T} \}$$

Integrable theory

$$w\eta^2 \frac{\partial^4}{\partial z^4} \log \mathcal{F}_n(z,w) + 6w\,\eta^2 \left(\frac{\partial^2}{\partial z^2} \log \mathcal{F}_n(z,w) \right)^2 + 2 \left(\frac{\partial}{\partial w} - \frac{\partial}{\partial z} \right) \log \mathcal{F}_n(z,w)$$
$$+ \left(\left[2(2n+\nu)\eta - 2z + w\,(1-\eta^2) \right] \frac{\partial^2}{\partial z^2} + 2(z - 2w) \frac{\partial^2}{\partial z \partial w} + 3w \frac{\partial^2}{\partial w^2} \right) \log \mathcal{F}_n(z,w) = 0$$

Quantum transport in chaotic cavities

Conductance-shot-noise power joint cumulants



conductance $g = \sum_{j=1}^{n} T_j$ and the shot-noise power $p = \eta \sum_{j=1}^{n} T_j(1 - T_j)$

Joint cumulant generation function

$$\log \mathcal{F}_n(z, w) = \sum_{\ell=1}^{\infty} (-1)^{\ell+m} \kappa_{\ell,m} \frac{z^\ell w^m}{\ell! m!}$$

The conductance cumulants are known, they play a role of initial condition for calculation of the shot-noise-power cumulants

$$\kappa_{\ell,0} \equiv \langle\langle g^\ell \rangle\rangle$$

Mean value

$$\kappa_{0,1} \equiv \langle\langle p \rangle\rangle = \eta \frac{n(n+\nu)}{2n+\nu} \left[1 + \frac{n(n+\nu)}{(2n+\nu)^2 - 1} \eta \right]$$

Variance

$$\kappa_{0,2} \equiv \langle\langle p^2 \rangle\rangle = \frac{\eta^2}{15} \left[2(2n+\nu)^2 \eta^2 \kappa_{4,0} + 15(2n+\nu)\eta \kappa_{3,0} + \left(15 + 3\eta^2\right) \kappa_{2,0} - 3\eta^2 (6\kappa_{2,0}{}^2 + \kappa_{4,0}) \right]$$

$$n \gg 1 \quad \nu = 0$$

$$\kappa_{0,m} \equiv \langle\langle p^m \rangle\rangle \simeq \eta^m \Bigg[\underbrace{2n \left(1 + \frac{\eta}{4}\right) \delta_{m,1} + \left(1 + \frac{\eta^2}{8}\right) \delta_{m,2}}_{\text{Gaussian part}} + \frac{(m-1)!}{8n^m} \left[\left(\frac{\eta}{2} - 1\right)^m + \left(\frac{\eta}{2} + 1\right)^m \right] \Bigg]$$

V.Al.Osipov, E.Kanzieper, “Statistics of thermal to shot noise crossover in chaotic cavities”, *J.Phys.A:Math.Theor.* **42** (2009) 475101

Averaged characteristic polynomials

The averaged characteristic polynomials appear in SUSY

$$\Pi_n\left(\{\,\varepsilon_\alpha;\kappa_\alpha\,\}_{\alpha=1}^{p}\right)\equiv\left\langle\prod_{\alpha=1}^{p}\det{}^{\kappa_\alpha}\left[\varepsilon_\alpha-\mathcal{H}\right]\right\rangle_{\mathcal{H}_{n\times n}}$$

The averaging is taken over ensemble of random Hermitian matrices with the Gaussian probability measure (GUE)

$$\langle\bullet\rangle_{\mathcal{H}}=c_n^{-1}\int_{\mathcal{H}^\dagger=\mathcal{H}}(\bullet)\,e^{-\mathrm{Tr}\,\mathcal{H}\mathcal{H}^\dagger}\prod_{j=1}^{n}d\mathcal{H}_{jj}\prod_{j<k}^{n}d\mathcal{H}_{jk}^{\mathrm{Re}}d\mathcal{H}_{jk}^{\mathrm{Im}}$$

Integrable theory

$$\left[\hat{\mathcal{B}}_{-1}^4+8(n-\kappa)\hat{\mathcal{B}}_{-1}^2-4(2\hat{\mathcal{B}}_0-3\hat{\mathcal{B}}_0^2+4\hat{\mathcal{B}}_1\hat{\mathcal{B}}_{-1})\right]\log\Pi_n+6\left(\hat{\mathcal{B}}_{-1}^2\log\Pi_n\right)^2=8n\kappa$$

$$\kappa=\sum_{\alpha=1}^{p}\kappa_\alpha \qquad \hat{\mathcal{B}}_q\equiv\sum_{\alpha=1}^{p}\varepsilon_\alpha^{q+1}\frac{\partial}{\partial\varepsilon_\alpha}$$

V.Al.Osipov, E.Kanzieper, "Correlations of RMT characteristic polynomials and integrability: Random Hermitian matrices", *Annals of Physics* **325** (2010) 2251

Averaged characteristic polynomials

$$\Pi_n\left(\{\varepsilon_\alpha;\kappa_\alpha\}_{\alpha=1}^p\right) \equiv \left\langle \prod_{\alpha=1}^p \det{}^{\kappa_\alpha}\left[\varepsilon_\alpha - \mathcal{H}\right] \right\rangle_{\mathcal{H}_{n\times n}}$$

Integrable theory

Multidimensional Painleve equation (?):

$$\left[\hat{\mathcal{B}}_{-1}^4 + 8(n-\kappa)\hat{\mathcal{B}}_{-1}^2 - 4(2\hat{\mathcal{B}}_0 - 3\hat{\mathcal{B}}_0^2 + 4\hat{\mathcal{B}}_1\hat{\mathcal{B}}_{-1})\right] \log\Pi_n + 6\left(\hat{\mathcal{B}}_{-1}^2\log\Pi_n\right)^2 = 8n\kappa$$

$$\kappa = \sum_{\alpha=1}^p \kappa_\alpha \qquad \hat{\mathcal{B}}_q \equiv \sum_{\alpha=1}^p \varepsilon_\alpha^{q+1}\frac{\partial}{\partial\varepsilon_\alpha}$$

For a single determinant model it reduces to Painleve IV:

$$\Pi_n(\varepsilon:\kappa) = \left\langle \det{}^{\kappa}\left[\varepsilon - \mathcal{H}\right]\right\rangle_{\mathcal{H}_{n\times n}}$$

$$\phi''' + 6(\phi')^2 - 4(\varepsilon^2 - 2(n-\kappa))\phi' + 4\varepsilon\phi - 8n\kappa = 0 \qquad \phi = \frac{\partial}{\partial\varepsilon}\log\Pi_n(\varepsilon;\kappa)$$

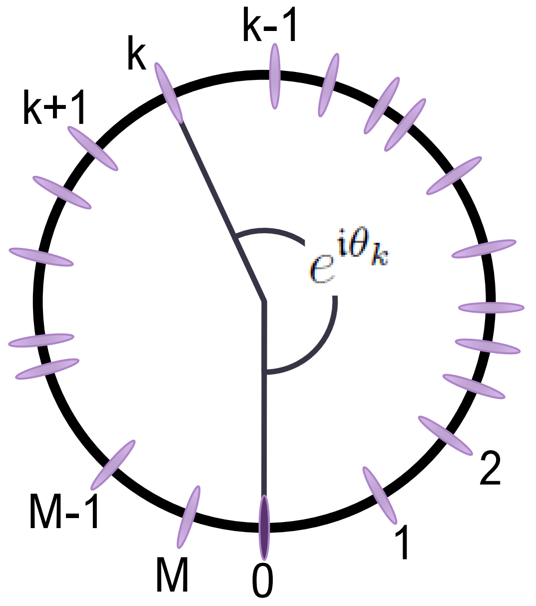
V.Al.Osipov, E.Kanzieper, "Correlations of RMT characteristic polynomials and integrability: Random Hermitian matrices", *Annals of Physics* **325** (2010) 2251

Random Matrix Theory.

Power spectrum of a chaotic system

- Power spectrum of a quantum chaotic system is expressed in terms of Painleve transcendent equation and behaves differently than 1/f-noise as it has been assumed earlier.

The eigenangles of a random Unitary matrix with a fixed zero:



The Joint Probability Density Function (Tuned CUE):

$$P_M(\boldsymbol{\theta}) = \frac{1}{M!} \prod_{k<\ell} \left| e^{i\theta_\ell} - e^{i\theta_k} \right|^2 \cdot \prod_{\ell=1}^{M-1} \left| 1 - e^{i\theta_\ell} \right|^2$$

Power spectrum: $\delta\theta_k = \theta_k - < \theta_k >$

$$S_M(\omega) = \frac{M}{4\pi^2} \sum_{\ell=1}^{M-1} \sum_{k=1}^{M-1} < \delta\theta_\ell \delta\theta_k > e^{i\omega(\ell-k)}$$

Integrable theory

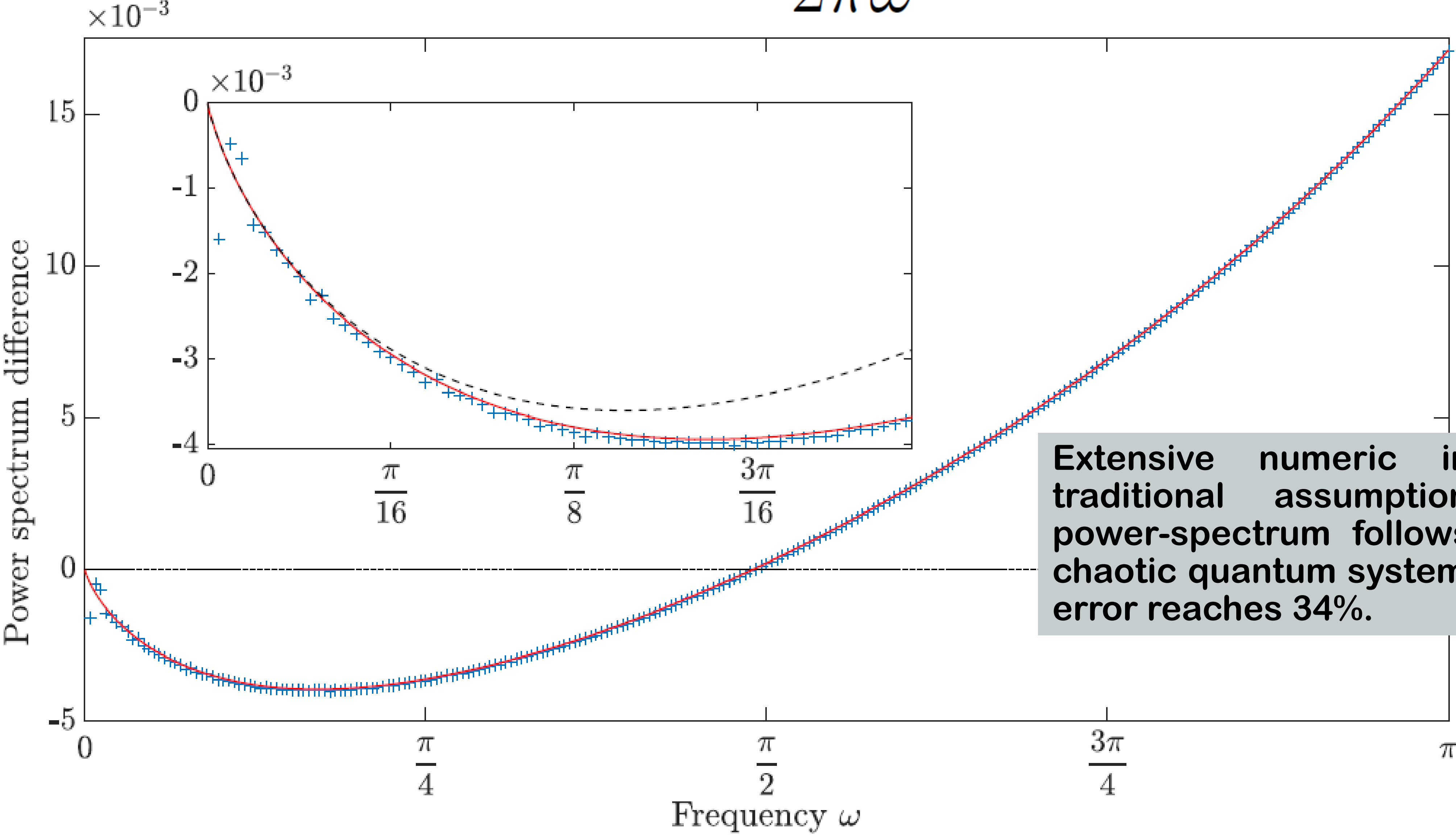
The power spectrum is expressed in terms of Painleve VI

The large-M asymptotic through Painleve V

R.Riser, **V.Al.Osipov**, E.Kanzieper, “Power-spectrum of long eigenlevel sequences in quantum chaology”, *Phys.Rev.Let.* **118** (2017) 204101
R.Riser, **V.Al.Osipov**, E.Kanzieper, “Nonperturbative theory of power spectrum in complex systems”, *Annals of Physics* **413** (2020) 168065

$$S_M(\omega) - \frac{1}{2\pi\omega}$$

Numerics: M=512
averaging over 4 x 10^8 matrices



Extensive numeric invalidates a traditional assumption that the power-spectrum follows 1/f law for chaotic quantum system. The relative error reaches 34%.

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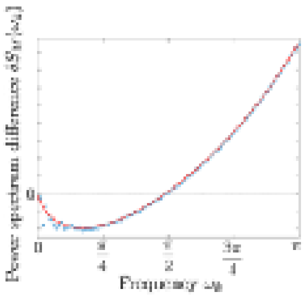


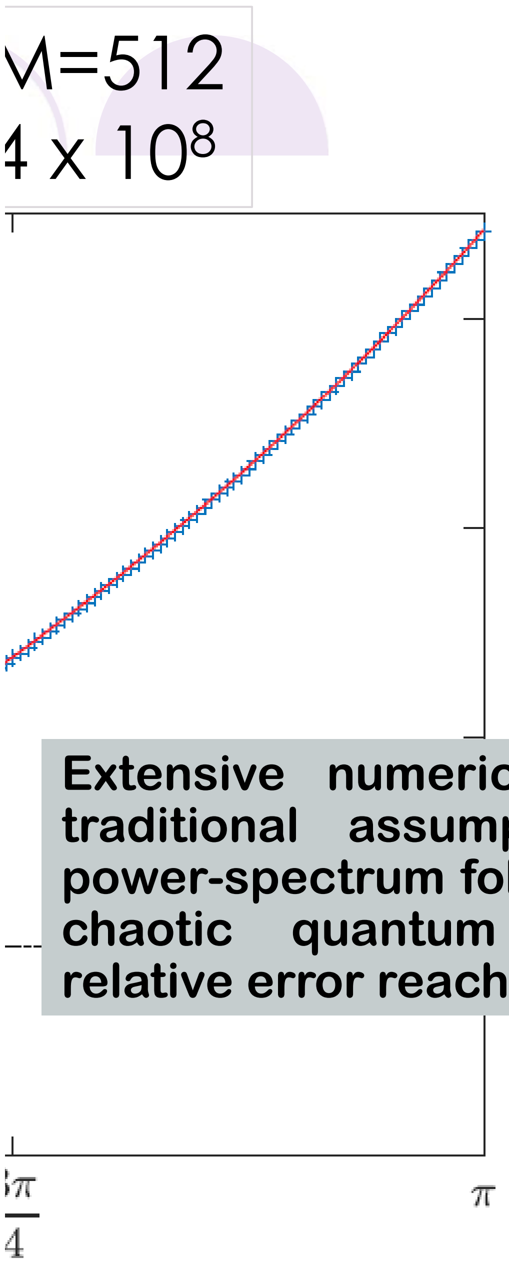
Power spectrum of long eigenlevel sequences in quantum chaotic systems

Roman Riser, Vladimir Al. Osipov, and Eugene Kanzieper
Phys. Rev. Lett. **118**, 204101 (2017)

Published 16 May 2017

has been highlighted by the editors as an Editors' Suggestion. Publication of a Letter is already a considerable achievement, as *Physical Review Letters* accepts fewer than 1/4 of submissions, and is ranked first among physics and mathematics journals by the Google Scholar five-year h-index. A highlighted Letter has additional significance, because only about one Letter in six is highlighted as a Suggestion due to its particular importance, innovation, and broad appeal. Suggestions are downloaded twice as often as the average Letter, and are covered in the press substantially more often. If Suggestions were a separate publication, they would have an Impact Factor of 13. More information about our journal and its history can be found on our webpage prl.aps.org.

Yours sincerely,



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Random Matrix Theory.
Applications to quantum transport and quantum chaos

The achievement has been reflected in press-relies of H.I.T and Lund University and in scientific mass-media:
http://www.hit.ac.il/sciences/news events/KanzieperResearchQuantum
http://www.lu.se/article/ekvation-satter-ngret-pa-kaos
https://futurism.com/new-equation-explains-quantum-chaos

HARD SCIENCE

New Equation Explains Quantum Chaos

We just found a new language for chaos theory.

CHELSEA GOHD | SEPTEMBER 4TH 2017

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Chaotic Equations

Jeff Goldblum's character (the
awareness to the general conc
highly-sensitive systems, able
And, while enigmatic in many
equation that answers the que

Theoretical But Practical

According to Vladimir Osipov, a researcher at Lund University's Faculty of Science and one of the study authors, "In chaotic quantum systems, the energy levels repel each other, and they affect each other even if they are far apart."

Scientists can now represent the chaotic behavior described by the math in a quantum system with perfect accuracy: "Yes, we now have an exact equation. Personally, I am actually surprised that it was

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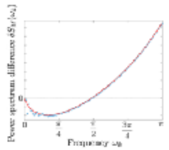

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Yours sincerely,

Pierre Meystre
Editor in Chief
Physical Review

...rs

...etters, 1 Research Road, Ridge, NY 11961-2701

Thank you

תודה רבה

Спасибо

谢谢您