Arithmetic in infinite extensions of number fields

Christian Maire
(joint work with ravi Ramakrishna (Cornell Univ.) and Farshid Hajir (Univ. of Massachusetts))

Abstract.

In this lecture, I will present recent results concerning arithmetic properties of infinite unramified extensions of number fields, as: (1) the new records for Martinet's constants; (2) an answer to a question of Ihara about the set of splitting; (3) the relationship with the p-rational fields. The talk will be elementary and accessible (without proof. . .).

An application of a conjecture of Mazur-Tate to supersingular elliptic curves

Emmanuel Lecouturier

Abstract.

In 1987, Barry Mazur and John Tate formulated refined conjectures of the "Birch and Swinnerton-Dyer type", and one of these conjectures was essentially proved in the prime conductor case by Ehud de Shalit in 1995. Combining the work of de Shalit and the theory of the Eisenstein ideal, we prove the following identity on supersingular j-invariants.

Let N be a prime number and $p \geq 5$ be a prime dividing $N-1$. Fix a surjective group homomorphism $\log : \mathbf{F}_{N^2}^{\times} \to \mathbf{Z}/p\mathbf{Z}$. Let $S = \{E_0, ..., E_g\}$ be the set of isomorphism classes of supersingular elliptic curves over $\bar{\mathbf{F}}_N$. We denote by $j(E_i) \in \overline{\mathbf{F}}_N$ the j-invariant of E_i; it is well-known that $j(E_i) \in \mathbf{F}_{N^2}$. Let $\mathcal{T}(S)$ be the set of spanning trees of the complete graph with vertices in S. If $T \in \mathcal{T}(S)$, let $E(T)$ be the set of edges of T. If $0 \leq i \neq j \leq g$, let $[E_i, E_j]$ be the edge between E_i and E_j. We have:

$$\sum_{T \in \mathcal{T}(S)} \prod_{[E_i,E_j] \in E(T)} \log(j(E_i) - j(E_j)) = 0 \ .$$

The $1/2$ Conjecture on q-binomial coefficients

Guoniu Han

Abstract. We introduce the super power series ring and the integer point order. We conjecture that the q-binomial coefficients take the maximal value with respect to the integer point order at the point $1/2$.

A combinatorial model for (q, y)-Laguerre polynomials

Jiang Zeng

Abstract. We consider a family of orthogonal polynomials, called (q, y)-Laguerre polynomials, whose moments are a q-generalization of Eulerian polynomials. It turns out that the latter is a rescaled version of Al-Salam-Chihara polynomials. We give a combinatorial interpretation of (q, y)-Laguerre polynomials by deforming a colored version of Foata-Strehl's Laguerre configurations. A combinatorial description of the corresponding moments are also given and the positivity of the linearization coefficients is proved. This a joint work with Qiong Qiong PAN.

Enumeration of simultaneous core partitions

Huan Xiong

Abstract. A partition is called a $(t_1, t_2, \ldots, t_m)$-core partition if it is simultaneously a t_1-core, a t_2-core, ..., a t_m-core partition. A number of methods, from several areas of mathematics, have been used in the study of t-core and $(t_1, t_2, \ldots, t_m)$-core partitions. In this talk, I will present several problems and conjectures on the enumeration of various simultaneous core partitions.

Positivity and divisibility of alternating descent polynomials

Zhicong Lin

Abstract. The alternating descent statistic on permutations was introduced by Chebikin as a variant of the descent statistic. We show that the alternating descent polynomials on permutations is unimodal via a five-term recurrence relation. We also found a quadratic recursion for the alternating major index q-analog of the alternating descent polynomials. As an interesting application of this quadratic recursion, we show that $(1+q)^{\lfloor n/2 \rfloor}$ divides $\sum_{\pi \in \mathfrak{S}_n} q^{\mathrm{altmaj}(\pi)}$, where $\mathfrak{S}_n$ is the set of all permutations of $\{1, 2, \dots, n\}$ and $\mathrm{altmaj}(\pi)$ is the alternating major index of π. Moreover, we study the γ-vectors of the alternating descent polynomials by using these two recursions and the **cd**-index. Further intriguing conjectures are formulated, which indicate that the alternating descent statistic deserves more work.