# 2024 泛函分析及空间理论天元暑期研讨班

## Concentration day:
## young researchers on noncommutative analysis

**July 31, 2024** **Zheng Xin Building, Room 23**

## 9:00-12:00

### Schatten properties of Calderon-Zygmund singular integral commutator on stratified Lie groups

***Fulin YANG***（杨福林） Harbin Institute of Technology

We provide full characterisation of the Schatten properties of $[M_b, T]$, the commutator of Calderón–Zygmund singular integral $T$ with symbol $b$ ($M_b f(x) := b(x)f(x)$) on stratified Lie groups $G$. We show that, when $p$ is larger than the homogeneous dimension $Q$ of $G$, the Schatten $S^p$ norm of the commutator is equivalent to the Besov semi-norm $B_p^{Q/p}$ of the function $b$; but when $p \leq Q$, the commutator belongs to $S^p$ if and only if $b$ is a constant. For the endpoint case at the critical index $p = Q$, we further show that the Schatten $S^{Q,\infty}$ norm of the commutator is equivalent to the Sobolev norm $W^{1,Q}$ of $b$. Our method at the endpoint case differs from existing methods of Fourier transforms or trace formula for Euclidean spaces or Heisenberg groups.

### Sharp constants in several inequalities on H-type groups

***Yaojun WANG***（王耀军） Fudan University

In this talk, I shall explain how to study a sharp functional inequality and review the research on sharp Hardy-Littlewood-Sobolev inequality on both Euclidean space and the Heisenberg group. I will use these two studies to illustrate the general procedure. At last, I will discuss the sharp fractional Sobolev inequality associated with fractional conformal subLaplacian $\mathcal{L}_s (0 < s < 1)$ on H-type groups. From these inequalities, we derive a sharp log-Sobolev inequality by considering a limiting case and a sharp Sobolev trace inequality. This is a joint work with Qiaohua Yang.

### Proper cocycles, measure equivalence and $L_p$-Fourier multipliers

***Gan YAO***（姚淦） Harbin Institute of Technology

We establish a new transference method of completely bounded $L_p$-Fourier multipliers for proper cocycles for pmp group actions on standard probability spaces. This generalizes the previous results by Haagerup and Jolissaint which only deals with the case $p = \infty$. In particular, this gives

a natural transference method of Fourier multipliers between groups with measure equivalence, which directly implies and notably generalizes the main result of Hong-Wang-Wang on the pointwise convergence of noncommutative Fourier series on amenable groups. As a second application, this theory also yields a transference method of $L_p$-Fourier multipliers from lattices in a linear Lie group to the whole group, which strengthens the previous results for Schur multipliers obtained by Haagerup and Lafforgue-de la Salle. As an example, we give a reasonable analog of Hilbert transform on $PSL_2(R)$ using the transference method. This is ongoing joint work with Simeng Wang.

### Stochastic theta methods for free stochastic differential equations

***Jiaxin WEI***（危嘉欣） Central South University

We introduce free probability analogues of the stochastic theta methods for free stochastic differential equations in this work. Under some mild conditions, we prove the strong convergence and exponential stability in mean square of the numerical methods. The free stochastic theta method with $\theta = 1$ can inherit the exponential stability of original equations for any given step size. Our method offers better stability and efficiency than the free Euler-Maruyama method. Moreover, numerical results are reported to confirm these theoretical findings.

## 14:00-17:40

### Interpolation of weak Orlicz types and the strong maximal in von Neumann algebras

***Jorge PÉREZ GARCÍA*** Instituto de Ciencias Matemáticas

The Marcinkiewicz interpolation theorem is a strong, well-known tool for classical analysts, which roughly states the following: an operator $T$ of weak type $(p_0, p_0)$ and $(p_1, p_1)$ is of strong type $(p, p)$ for all $p$'s in between $p_0$ and $p_1$. One of the strengths of this theorem is that the same proof works when $T$ is a sublinear operator - and, as a particular but relevant case, for $T$ a maximal operator. For non-commutative spaces, the proof becomes much more involved: the maximal version of the theorem in this context was originally proved by Junge and Xu in 2003, and just last year Cadhilac and Ricard found a simplified proof much closer to the classical one.

In joint work with Adrián González-Pérez and Javier Parcet, we adapt this last proof in order to work for maximal operators with weak Orlicz types at the extreme points. The quantitative bound that we obtain for the growth of the norm solves an open question about certain maximal operators in von Neumann algebras, which we will introduce and explain in detail.

## Maximal inequalities for noncommutative lacunary discrete spherical averages

***Qi SUN***（孙奇） Central South University

In this talk, we will discuss some recent progress in the study of noncommutative discrete spherical averages defined on a lacunary sequence. Specifically, we estalisheh the maximal inequality for noncommutative lacunary discrete spherical averages when the dimension $d > 3$. Our methods combine the Kloosterman refinement for the Fourier transform of spherical measures introduced by Magyar [J. Number Theory, 2007] with the interpolation inequality techniques developed by Cook and Hughes [Trans. Am. Math. Soc., 2021].

## Modulated (C,α)-ergodic theorems in noncommutative Lp-spaces

***Xin ZHANG***（张鑫） Wuhan University

In this paper, we assume $\alpha > 0$ and establish the weighted $(C, \alpha)$-maximal ergodic inequalities in noncommutative $L_p$ spaces for $1 \leq p \leq \infty$, drawing inspiration from works from Junge/Xu and Yeadon. This is done with a positive Dunford-Schwartz operator $T$ acting on a semifinite von Neumann algebra $\mathcal{M}$. Utilizing this result as a foundation, we then present the noncommutative Besicovitch weighted $(C, \alpha)$-ergodic theorems. These individual ergodic theorems can be partially viewed as a noncommutative extension of Yoshimoto's work in 2020, as our conclusions and conditions are more general. This appears to be new even in the commutative setting.

## Lunar monoid and the self-absorption property of its Hankel system

***Lijun WANG***（王李俊） Wuhan University

In recent work of Yong HAN, Yanqi QIU and Wangzi PENG, they introduce a notion of self-absorption property. They prove that Hankel system on lunar monoids admits self-absorption property, which is applied to the study of completely bounded Fourier multipliers between Hardy spaces. In this talk, we will show some recent progress in the study of relation between lunar condition on monoids and the self-absorption property of its Hankel system. The talk is based on joint work with Yong HAN, Yanqi QIU and Wangzi PENG.

## The nonlinear estimates on quantum Besov space

***Deyu CHEN***（陈德宇） Harbin Institute of Technology

In this talk, we will firstly review the relevant results and methods of the superposition operator on classical Besov space; then talk about the analogue for Meyer's method in the noncommutative case. Finally we present the progresses what we have made in this direction, in particular the nonlinear estimates for the superposition operator with a non-smooth symbol on quantum Besov space. We will also introduce the theory about multiple operator integral which is used to overcome the difficulties caused by the noncommutativity.